Price Level and Inflation Dynamics in Heterogeneous Agent Economies

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August 22, 2023

Abstract

We study price level dynamics in a heterogeneous-agent incomplete-market economy with nominal government debt and flexible prices. Unlike in representative agent economies, steady-state equilibria exist when the government runs persistent deficits, provided the level of deficits is not too large. We quantify the maximum sustainable deficit for the US and show that it is lower under more redistributive tax and transfer systems. With constant primary deficits, there exist two steady-states, and the price level and inflation are not uniquely determined. We describe alternative policy settings that deliver uniqueness. We conduct quantitative experiments to illustrate how redistribution and precautionary saving amplify price level increases in response to fiscal helicopter drops, deficit expansions, and loose monetary policy. We show that rising primary deficits can account for a decline in the long-run real interest rate, leading to permanently higher inflation. Our work highlights the role of household heterogeneity and market incompleteness in determining inflation dynamics.

Keywords: Fiscal theory of the price level, Heterogeneity, Incomplete markets, Inflation, Precautionary saving, Redistribution, Sustainable deficit.

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We are grateful to Mark Aguiar, Fernando Alvarez, Marco Bassetto, Pierpaolo Benigno, John Cochrane, Joel P. Flynn, Mariano Kulish, Chen Lian, Francesco Lippi, Sebastian Merkel, Ben Moll, Bruce Preston, Thomas Sargent, and Chris Sims for helpful comments. First version: 28 April 2023.

1 Introduction

We develop a framework to study the causes and consequences of price level dynamics in an economy with three features: (i) a fiscal authority issues nominal debt to finance committed real expenditures and transfers to households; (ii) a monetary authority sets the short-term nominal rate on government debt;(iii) financial markets are incomplete, so households have a precautionary motive to accumulate savings in order to self-insure against idiosyncratic income risk.

Our interest in economies with the first two features is motivated by institutional arrangements in the real world. Such economies have been extensively studied, most recently under the label *Fiscal Theory of the Price Level* (FTPL).[1] They have also been a useful lens to analyze the most recent bout of inflation that followed large expansions in government borrowing, a global supply shock due to the COVID-19 pandemic, and sharp interest rate movements by central banks around the world. This literature has focused almost entirely on representative agent economies.

Our motivation for extending this analysis to "Bewley" economies (Bewley, 1987) with heterogeneous agents and incomplete markets is three-fold. First, heterogeneous agent models generate consumption responses to income and interest rates that are consistent with the vast body of micro-economic evidence on the joint dynamics of household income and spending.[2] This property is important because household spending pressure is a key force shaping inflation and interest rates in equilibrium.

Second, household heterogeneity has played an important role in both the drivers and consequences of the current inflationary episode. Governments issued vast quantities of new debt to finance transfers that were targeted to certain groups of households. The ongoing spending pressures that are leading many government to run persistent deficits are also highly targeted. Quantitative heterogeneous agent models are a natural environment to study the implications of such interventions, as well as the distributional effects of shocks and subsequent policy responses.

[1]The FTPL literature, which has its roots in Sargent and Wallace (1981) and builds on Leeper (1991), Sims (1994), Woodford (1995) and Cochrane (1998) is too vast to cite in full. See the handbook chapter by Leeper and Leith (2016) and book by Cochrane (2023) for a synthesis of the reach of FTPL models.

[2]See for example the review article by Kaplan and Violante (2022).

Third, working in a heterogeneous-agent incomplete-market setting also overcomes a limitation of representative agent FTPL models that makes their application to current macroeconomic conditions problematic. Standard representative agent models require governments to run positive primary surpluses in expectation at all points in time. However, in recent decades the US has consistently run primary deficits, and the fiscal positions of the US and many other developed economies look unlikely to return to surpluses anytime soon.[3] Heterogeneous agent versions of these models offer a natural setting in which to study price level dynamics with persistent primary deficits. In these versions, the real return on government debt r is less than the growth rate of the economy g, which is also a feature of recent macroeconomic conditions.

This motivation leads us to start building a bridge between the well-studied representative-agent FTPL and workhorse heterogeneous-agent models in the tradition of Bewley (1987), Imrohoroğlu (1989), Huggett (1993) and Aiyagari (1994). In this paper, we take a first step by focusing on flexible-price economies.[4]

Theoretical Analysis. We begin by analyzing an endowment economy in which the government runs positive primary surpluses and $r > g$. Here, the conditions on monetary and fiscal policy for the price level and inflation to be uniquely determined are essentially unchanged from corresponding representative agent economies. There are, however, important quantitative differences that reflect the role of precautionary savings. Unlike in the representative agent economy, in the heterogeneous agent economy changes in fiscal policy lead to movements in the real interest rate. This is because a change in either the level of debt, or the size and distribution of surpluses alters the overall demand for savings among households. For a given setting of monetary policy, these different real rate dynamics imply different paths of inflation. It also means that there are non-trivial inflation dynamics following a one-time fiscal helicopter drop, and that the path of inflation depends on the targeting of the fiscal injection. We use a modified representative agent model with bonds in the

[3]With the exception of 1998-2001, the US has not run a primary surplus since 1970. See Series FYFSD from FRED, Federal Reserve Bank of St. Louis, https://fred.stlouisfed.org. Moreover, the May 2023 10-year budget projections of the Congressional Budget Office (CBO) estimate that deficits will remain negative at least until 2033: https://www.cbo.gov/data/budget-economic-data

[4]In ongoing work we extend to economies with nominal rigidities. See Kaplan et al. (2023).

utility function to provide intuition for these forces. We then analyze the same heterogeneous-agent economy but with a government that runs a constant primary deficit and $r < g$. We show that, as long as the level of deficits is not too large, equilibria with a finite price level where debt is valued exist. The maximum possible level of deficits is decreasing in the amount of redistribution implicit in the tax and transfer system: more redistribution reduces aggregate precautionary saving and increases real interest payments on debt. For lower levels of deficits, there are generically two steady-states. Thus, without additional assumptions, standard FTPL arguments do not uniquely pin down the price level or the path of inflation. The steady-states are Pareto ranked, with the high debt, high interest rate, low inflation steady-state delivering larger welfare to every household. The low inflation steady-state is saddle-path stable: there is a unique initial price level and subsequent path of inflation and real rates leading to that steady-state. The high inflation steady-state is locally stable: there is a continuum of initial price levels that support paths of inflation leading to that steady-state.

We discuss various extensions that deliver a unique prediction for the price level and inflation. First, we propose modifications to the model that eliminate the high inflation steady-state altogether, leaving only a unique saddle-path stable steady-state. These modifications include (i) fiscal reaction rules that allow the level of surpluses to respond to deviations of real debt or the real rate from steady-state; and (ii) the introduction of a foreign sector with a relatively inelastic demand for domestic government debt. Second, we propose a policy environment in which the central bank successfully coordinates private sector expectations about long-run inflation. By anchoring long-run inflation expectations to be consistent with the saddle-path stable steady-state, uniqueness is also achieved in the short run, because all the equilibria that converge to the high inflation steady-state are eliminated.

With uniqueness of equilibria in hand, we move to the quantitative analysis.

Quantitative Policy Messages. In the quantitative part of the paper, we conduct a series of experiments to illustrate lessons for policy that emerge in the heterogeneous agent setting, but are concealed in more traditional representative agent FTPL environments.

First, we consider the effects of permanently increasing deficits. We calculate that if the government were to permanently increase lump sum transfers to households without raising taxes, the largest sustainable primary deficit would be 4.6% of GDP, or 40% higher than current levels. The maximum sustainable deficit depends on the degree of social insurance: expanding deficits in a more progressive manner implies lower maximum deficits. The reason is that tax systems that provide more social insurance weaken precautionary savings, thus lowering household demand for government debt. More progressive tax systems therefore reduce fiscal space.

A permanently higher deficit is associated with a lower steady-state real interest rate and less real government debt, as well as a higher long-run inflation rate for a given nominal rate target. This is because a larger deficit must be funded by larger real interest receipts, which require a more negative real rate. The heterogeneous agent framework thus offers an alternative interpretation of discussions around secular stagnation by highlighting the connection between a rising primary deficit, falling real rates and rising inflation.

Next, we study the effects of issuing new debt while holding primary deficits constant: a fiscal helicopter drop. We consider a helicopter drop of around 16% of annual GDP, roughly the size of the fiscal expansion in the US over the course of the COVID-19 pandemic. Consistent with the representative agent experiments in Cochrane (2022), we find that this generates an immediate jump in the price level. However, relative to the representative agent benchmark, in our economy there is an additional 30% initial increase in the price level. This amplification is driven by redistribution and heterogeneity of marginal propensities to consume (MPC): in the heterogeneous agent economy, the dilution of nominal debt entails large amounts of redistribution from wealthy to poor households. This reallocation of wealth generates upward pressure on consumption, which increases real rates and interest payments on government debt, thereby causing a larger initial jump in the price level. A targeted helicopter drop such as that implemented in the US, which targets high MPC households, fuels additional short-term inflationary pressures.

Lastly, we study the effects of purely redistributive policies that hold both debt and deficits constant, and show that budget neutral redistribution is inflationary. We illustrate these effects by way of numerical experiments in which the government levies a one-time wealth tax on household in the top percentiles of the wealth distribution, and redistributes the proceeds lump-sum to households in the bottom half of the wealth distribution. As with the fiscal helicopter drop, real redistribution towards high MPC households leads to a temporarily higher real interest rate and a downward revaluation of real assets through a jump in the price level.

Related Literature. Our paper belongs to a small but growing literature that moves beyond the representative agent model and explores the FTPL with incomplete markets. Bassetto and Cui (2018) show that a model of overlapping generations and a model in which government debt provides special liquidity services can give rise to multiple steady-states in which the real interest rate on government debt is below the growth rate of output. They emphasize that the FTPL can fail to yield price level determinacy in these settings. Brunnermeier et al. (2020, 2022), Miao and Su (2021) and Amol and Luttmer (2022) all study models with idiosyncratic risk in the rate of return on capital, and explore settings for fiscal policy that can establish price level uniqueness in low interest rate environments.

Our work differs from these papers in three respects. First, we investigate the implications of the FTPL in a Bewley (1987) economy in which market incompleteness arises from uninsurable labor income risk.[5] In doing so, we emphasize the importance of MPC heterogeneity in driving price level and inflation dynamics. Second, we explore a wide class of fiscal, monetary, and institutional specifications and show how they lead to price level uniqueness in models where the government runs persistent primary deficits. Third, we quantitatively explore the response of economic aggregates to unanticipated shocks in low-interest rate economies with persistent deficits. To the best of our knowledge, the messages we deliver about the role of precautionary savings and MPC heterogeneity in driving price level, inflation and real rate dynamics in this class of economies are novel.[6]

Our work also relates to the literature that studies the implications of low interest

[5]Hagedorn (2021) also explores price-level determination in a "Bewley" economy with nominal government debt, but focuses on a different class of fiscal policies outside FTPL.

[6]Some qualitative aspects of our analysis, such as equilibrium multiplicity with deficits, share features with certain monetarist economies. See, for example, Chapter 18 of Ljungqvist and Sargent (2018).

rate environments for government borrowing (Aguiar et al., 2021; Blanchard, 1985, 2019; Cochrane, 2021; Kocherlakota, 2023; Mehrotra and Sergeyev, 2021; Reis, 2021). This body of work emphasizes that the government can roll over debt indefinitely when the real interest rate on government debt is below the growth rate of the economy.[7] We show that this stark conclusion is correct only up to a limit: there is a finite upper bound on primary deficits for there to exist an equilibrium in which government debt is valued. We quantify this bound in our calibrated model for the U.S. economy and illustrate how it depends on the level of uninsurable income risk and on the degree of fiscal redistribution.[8]

Finally, our work highlights the importance of household heterogeneity in determining interest rates and inflation. As such, it relates to work that explores the distributional consequences of monetary policy and inflation (Doepke and Schneider, 2006; Coibion et al., 2017; McKay and Wolf, 2023) and the role of agent heterogeneity in amplifying economic outcomes (Auclert et al., 2018; Kaplan et al., 2018; Auclert, 2019). In particular, we show that unanticipated changes in the price level can give rise to non-trivial, persistent dynamics in the real interest rate and inflation due to heterogeneous wealth effects across the distribution.

2 Model Environment

2.1 Households

Demographics. Time is continuous and is indexed by $t \geq 0$. The economy is populated by a continuum of households indexed by $j \in [0,1]$.

Endowments. Real aggregate output y_t is exogenous and grows at a constant rate $g \geq 0$. Household j receives a stochastic share z_{jt} of aggregate output. The shares z_{jt} are independent across households and a law of large numbers holds so that there is no economy-wide uncertainty,

$$\int_{j\in[0,1]} z_{jt}\mathrm{d}j = 1 \text{ for all } t \geq 0. \tag{1}$$

[7] Angeletos et al. (2023) show that in non-Ricardian economies with nominal rigidities, it is possible for government deficits to be self-financing, even when $r > g$.

[8] The insight that the size of fiscal space depends on the use the government makes of this space is shared by Mian et al. (2021a) and Amol and Luttmer (2022). However, precautionary saving plays no role in the two-agent model of Mian et al. (2021a), and redistribution plays no role in the model of Amol and Luttmer (2022) where all agents have the same MPC. In our economy, the strength of consumption insurance and fiscal redistribution forces determined endogenously in equilibrium.

In our baseline model we assume that z_{jt} follows an N-state Poisson process with switching intensities $\lambda_{z,z'}$. The lowest value of the endowment share $\underline{z}$ is strictly positive, $\underline{z} > 0$, from which it follows that the natural borrowing limit is below zero.[9] Not For Publication Appendix G presents a model in which z_{jt} follows a diffusion.

Assets. Households trade a short-term risk-free bond that yields a nominal flow return i_t. We denote the nominal bond holdings of household j at time t by A_{jt}. This asset is the unit of account in the economy, and we let P_t denote the price of output in terms of this short-term bond.

Preferences. Households take the path of aggregate variables $\{P_t, i_t, y_t\}_{t\geq 0}$ as given and choose real consumption flows $\tilde{c}_{jt}$ to maximize

$$\mathbb{E}_0 \int e^{-\tilde{\rho} t} \frac{\tilde{c}_{jt}^{1-\gamma}}{1-\gamma} \mathrm{d}t \tag{2}$$

with $\gamma \geq 0$, where the expectation is taken over the idiosyncratic endowment process z_{jt}. We denote the household's discount rate by $\tilde{\rho} > 0$.

Nominal Household Budget Constraint. Initial nominal assets A_{j0} are given. For $t > 0$, households face a flow budget constraint

$$\mathrm{d}A_{jt} = \left[i_t A_{jt} + \left(z_{jt} - \tau_t(z_{jt})\right) P_t y_t - P_t \tilde{c}_{jt}\right] \mathrm{d}t. \tag{3}$$

The path of tax and transfer functions $\tau_t(z)$ is set by the fiscal authority and is described in more detail below. Nominal savings dA_{jt} are equal to the sum of asset income $i_t A_{jt}$ and endowment income net of taxes and transfers $\left(z_{jt} - \tau_t(z_{jt})\right) P_t y_t$, minus consumption expenditures $P_t \tilde{c}_{jt}$. In our baseline model we assume that households cannot borrow $A_{jt} \geq 0$, but we relax this assumption in Section 5. Online Appendix E.1 contains an analysis of the model with borrowing.

Price Level and Inflation. Since this is a flexible-price economy, the price level P_t may exhibit jumps. For ease of notation and exposition, we restrict the price level to jump only at $t = 0$, after which it follows a deterministic path.[10] Since there is no

[9] In our quantitative experiments in which we allow for borrowing, the interest rate on loans is always positive so the natural debt limit is well-defined.

[10] Studying perfect foresight solutions with a single probability-zero jump at time zero is commonly maintained in FTPL models (Leeper, 1991; Sims, 2011; Cochrane, 2018). The absence of aggregate uncertainty implies that the price level cannot exhibit jumps for $t > 0$ in discrete time, representative

intrinsic (i.e., fundamental) aggregate uncertainty, this implies perfect foresight over aggregate variables for $t > 0$. For $t > 0$, we define the inflation rate by

$$\frac{\mathrm{d}P_t}{P_t} = \pi_t \mathrm{d}t. \tag{4}$$

De-trended Real Household Budget Constraint. We denote de-trended real assets and de-trended real consumption as

$$a_{jt} := \frac{A_{jt}}{P_t y_0 e^{gt}} \qquad c_{jt} := \frac{\tilde{c}_{jt}}{y_0 e^{gt}} \tag{5}$$

For $t > 0$, we can re-write the nominal budget constraint (3) in de-trended real terms:

$$\mathrm{d}a_{jt} = \left[r_t a_{jt} + z_{jt} - \tau_t(z_{jt}) - c_{jt}\right] \mathrm{d}t \tag{6}$$

where

$$r_t := i_t - \pi_t - g \tag{7}$$

is the growth-adjusted real rate. At $t = 0$, de-trended real assets a_{j0} are given by the ratio of initial nominal assets A_{j0} to the endogenous initial price level P_0.

Relative Asset Holdings. Let A_t and a_t denote aggregate nominal and aggregate de-trended real household assets, respectively:

$$A_t := \int_{j\in[0,1]} A_{jt} \mathrm{d}j \qquad a_t := \int_{j\in[0,1]} a_{jt} \mathrm{d}j$$

We denote the share of assets held by household j at time t by $\omega_{jt} := \frac{A_{jt}}{A_t} = \frac{a_{jt}}{a_t}$, with

$$\int_{j\in[0,1]} \omega_{jt} \mathrm{d}j = 1 \text{ for all } t \geq 0. \tag{8}$$

Recursive Formulation of Household Problem. Given paths of real rates r_t and taxes τ_t, the household problem can be expressed recursively via the Hamilton-Jacobi-Bellman Equation (HJB)

$$\begin{aligned} \rho V_t(a,z) - \partial_t V_t(a,z) &= \max_c \frac{c^{1-\gamma}}{1-\gamma} + \partial_a V_t(a,z)\left[r_t a + z - \tau_t(z) - c\right] \\ &\quad + \sum_{z' \neq z} \lambda_{z,z'} \left[V_t(a,z') - V_t(a,z)\right], \end{aligned} \tag{9}$$

agent FTPL models (Cochrane, 2023).

together with the boundary condition $\partial_a V_t(0,z) \geq (z - \tau_t(z))^{-\gamma}$ that ensures that the borrowing constraint $a \geq 0$ is satisfied. The growth-adjusted discount rate ρ in (9) is defined as $\rho = \tilde{\rho} - (1-\gamma)g$.

The optimal consumption function $c_t(a,z)$ that solves the HJB is defined by

$$c_t(a,z) = \left[\partial_a V_t\left(a,z\right)\right]^{-\frac{1}{\gamma}}. \tag{10}$$

The associated savings function is denoted by

$$\varsigma_t(a,z) := r_t a + z - \tau_t(a,z) - c_t(a,z) \tag{11}$$

If a value function $V_t(a,z)$ solves the HJB (9) and satisfies the boundedness condition

$$\lim_{T\to\infty} \mathbb{E}_T\left[e^{-\rho T} V_T(a_{jT}, z_{jT})\right] = 0, \tag{12}$$

then the stochastic process for consumption defined by (10) solves the sequence version of the household problem (2).[11]

The distribution of households across real asset holdings and endowment shares $g_t(a,z)$ satisfies the Kolmogorov Forward Equation (KFE)

$$\partial_t g_t(a,z) = -\partial_a\left[g_t(a,z)\varsigma_t(a,z)\right] - g_t(a,z)\sum_{z'\neq z}\lambda_{z,z'} + \sum_{z'\neq z}\lambda_{z',z} g_t(a,z'). \tag{13}$$

Let $f_t(\omega,z)$ denote the distribution of households across asset and endowment shares. For a given path of aggregate real wealth a_t, $f_t(\omega,z)$ and $g_t(a,z)$ are related by

$$f_t(\omega,z) = g_t(\omega a_t, z). \tag{14}$$

The KFE is a backward-looking equation where the initial distribution $g_0(a,z)$ is given.

2.2 Government

Nominal Government Budget Constraint. We assume a fiscal authority that issues short-term nominal government debt B_t subject to the budget constraint:

[11] See Theorem 3.5.3 in Pham (2009). The expectation in (12) is with respect to the stochastic process for idiosyncratic income and assets for household j, given by the budget constraint (6).

$$\mathrm{d}B_t = [i_t B_t - s_t P_t y_t]\,\mathrm{d}t \tag{15}$$

where s_t is the ratio of primary surpluses to output and is determined by the tax and transfer function as

$$s_t = \int_{j\in[0,1]} \tau_t\left(z_{jt}\right)\mathrm{d}j \tag{16}$$

Equation (15) defines the evolution of nominal government debt. This is a backward-looking equation where the initial level of nominal government $B_0 > 0$ is given. We restrict $B_t \geq 0$ so that the government can only borrow and not lend.[12]

De-trended Real Government Budget Constraint. We denote de-trended real government debt (or the debt-output ratio) by b_t,

$$b_t = \frac{B_t}{P_t y_0 e^{gt}}. \tag{17}$$

For $t > 0$, real debt b_t evolves according to the real version of the government budget constraint given by (15):

$$\mathrm{d}b_t = [r_t b_t - s_t]\,\mathrm{d}t. \tag{18}$$

Real debt increases whenever real interest rate payments exceed real primary surpluses. At $t = 0$, de-trended real debt b_0 is a jump variable given by the ratio of exogenously given initial nominal debt B_0 to the endogenous initial price level P_0.

Fiscal Policy. For our baseline analysis we focus on a time-invariant tax and transfer function $\tau_t(z) = \tau^*(z)$, so that surpluses or deficits are a constant fraction of real output $s_t = s^*$. In Section 4.3, we generalize the analysis to allow for a broader class of fiscal rules of the form

$$s_t = s(b_t, r_t). \tag{19}$$

These rules allow primary surpluses to respond to real aggregate debt, real interest rates or real interest payments and play an important role in determining the price level when governments run persistent deficits, $s_t < 0$.

Monetary Policy. For our baseline analysis we focus on a nominal interest rate peg $i_t = i^*$. In our quantitative analysis in Section 5 we allow for long-term debt

[12] Introducing government consumption would be subsumed in s_t in Equation (15), thereby leaving the key mechanisms of our model unchanged.

and a richer class of Taylor-type rules for nominal interest rates. We also discuss how allowing for other monetary rules affects our results about the determination of the price level and inflation in Section 2.3.

2.3 Equilibrium

We first define a *real equilibrium* as a collection of real variables which satisfy household optimality, are consistent with their laws of motion, and obey market clearing.

Definition 1. *Given (i) a constant tax and transfer function $\tau^*(z)$; and (ii) an initial distribution of households across asset and endowment shares $f_0(\omega, z)$, a real equilibrium is a collection of variables:*

$$\{V_t(a,z), c_t(a,z), f_t(\omega,z), g_t(\omega a_t, z), a_t, b_t, r_t\}_{t\geq 0} \tag{20}$$

such that, for all $t \geq 0$:

1. *the value function $V_t(a,z)$ solves the HJB* (9) *and satisfies the boundedness condition* (12)
2. *the consumption function is defined by* (10)
3. *the distribution of asset levels $g_t(\omega a_t, z)$ solve the KFE* (13)
4. *the distribution of household endowment shares $f_t(\omega, z)$ satisfies* (14)
5. *the path of government debt b_t satisfies the government budget constraint* (18)
6. *the asset market clears, $a_t = b_t$*

Note that by Walras' law, asset market clearing implies that the goods market clearing condition is also satisfied:

$$\int_{j\in[0,1]} c_{jt}\mathrm{d}j = 1 \text{ for all } t \geq 0.$$

Price Level and Inflation Determination. Under our assumptions about monetary and fiscal policy, each real equilibrium implies a unique initial price level P_0 and a subsequent unique path of inflation π_t. These are determined as follows. Each real equilibrium contains an initial value of real government debt b_0. Since initial nominal debt B_0 is given, the initial price level is determined as

$$P_0 = \frac{B_0}{b_0}.$$

The path of inflation is uniquely determined by the equilibrium path of real rates r_t and the nominal rate i^* which is set by the monetary authority as

$$\pi_t = i^* - r_t - g.$$

It follows that uniqueness of uniqueness of a real equilibrium implies uniqueness of the price level. If there is more than one real equilibrium then there will be more than one possible path for the price level. But if the real equilibrium is unique, then there is only one possible path for the price level P_t for $t \geq 0$, which is determined by initial nominal debt and monetary policy. As a result, we focus most of our analysis on the existence and uniqueness of real equilibria, with the understanding that whenever the real equilibrium is unique, so too is the price level and inflation.

Monetary Policy Rules. With flexible prices, the equivalence between uniqueness of real equilibria and uniqueness of the path of prices does not depend on our assumption of a nominal interest rate peg $i_t = i^*$. If the monetary authority instead follows an instantaneous feedback Taylor Rule of the form

$$i_t = i^* + \phi_m(\pi_t - \pi^*) \tag{21}$$

then inflation is uniquely determined as

$$\pi_t = \frac{i^* - \phi_m \pi^* - r_t - g}{1 - \phi_m}.$$

If the monetary authority follows a lagged feedback Taylor Rule of the form

$$\mathrm{d}i_t = -\theta_m \left[i_t - i^* - \phi_m(\pi_t - \pi^*)\right] \mathrm{d}t \tag{22}$$

then initial inflation is determined as $\pi_0 = i_0 - r_0 - g$ and subsequent inflation is determined as the unique forward solution to the ordinary differential equation

$$\mathrm{d}\pi_t = -\theta_m \left[\pi_t - \phi_m(\pi_t - \pi^*) + r_t - (g - i^*)\right] \mathrm{d}t - \mathrm{d}r_t.$$

Depending on parameter configurations, prices and inflation may not remain bounded, but there is nothing in the equilibrium definition that rules out such paths.

3 Primary Surpluses $s^* > 0$

We start by showing uniqueness of equilibrium when the fiscal authority runs positive primary surpluses. We use an example to illustrate the different dynamics in the heterogeneous agent economy compared to its representative agent counterpart.

3.1 Stationary Equilibrium

Household Asset Demand. In a stationary equilibrium, the real rate r_t is constant. Under regularity conditions that are well understood, with a constant interest rate r and transfer function $\tau^*(z)$, the solution to (9) and (13) implies a unique stationary distribution $g(a, z; r)$.[13] We use this result to construct a function $\mathbf{a}(r)$ that maps different interest rates into the aggregate quantity of real assets held by households in the corresponding stationary distribution,

$$\mathbf{a}(r) := \int_{a,z} a g(a, z; r) \mathrm{d}a \mathrm{d}z$$

It is well known that $\lim_{r \to \rho} \mathbf{a}(r) = \infty$. In addition we will assume that the function $\mathbf{a}(r)$ is continuous, differentiable and strictly increasing.[14] In Online Appendix C.2 we show that there exists an interest rate $\underline{r} < 0$ below which households do not hold any assets in the stationary distribution, so that $\mathbf{a}(r) = 0$ for all $r \leq \underline{r}$. The blue line in Figure 1 labelled $\mathbf{a}(r)$ is an example of a typical stationary asset demand function.

Government Asset Supply. In a stationary equilibrium, the government budget constraint defines a steady-state asset supply function $\mathbf{b}(r)$. This is obtained by setting $\mathrm{d}b_t = 0$ in (15),

$$\mathbf{b}(r) = \frac{s^*}{r}. \tag{23}$$

Since $b_t \geq 0$, this supply function takes the shape of a downward-sloping hyperbola in the positive quadrant as illustrated by the red line labelled $\mathbf{b}(r)$ in Figure 1.

Stationary Equilibrium. A stationary equilibrium requires that $\mathbf{a}(r) = \mathbf{b}(r)$, so that the asset market clears. Given our assumptions, there is a unique stationary real equilibrium shown as (b^*, r^*) in Figure 1. The assumption that primary surpluses are positive $s^* > 0$ implies that the stationary equilibrium real rate r^* is positive.

[13] See e.g. Bewley (1995), Stokey et al. (1989), and Aiyagari (1994).
[14] Achdou et al. (2022) show that sufficient conditions for this to be true are $\gamma \leq 1$ and $\underline{a} \geq 0$.

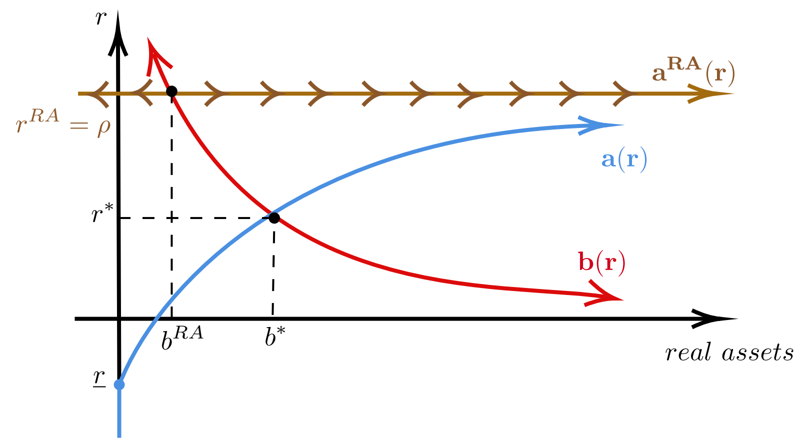


Figure 1: Steady state equilibrium with positive surpluses

The unique stationary equilibrium in the corresponding representative agent economy is the point (b^{RA}, r^{RA}) in Figure 1. In this economy the household asset demand curve is perfectly elastic at $r = \rho$. As is well known, in the heterogeneous agent economy the real rate is lower and the level of real government debt is higher than in the representative agent economy.

3.2 Non-Stationary Equilibrium

Because there is a unique stationary real equilibrium, in order to pin down the price level and inflation it suffices to rule out multiplicity of non-stationary real equilibria. Before tackling the heterogeneous agent economy, it is useful to recap the argument in the representative agent economy.

Uniqueness in Representative Agent Economies. In a representative agent economy, consumption satisfies an Euler equation of the form

$$\frac{\mathrm{d}c_t}{c_t} = \frac{1}{\gamma}\left(r_t - \rho\right)\mathrm{d}t$$

In an endowment economy, goods market clearing implies $\mathrm{d}c_t = 0$ and hence in equilibrium $r_t = \rho$ at all points in time, not just in a stationary equilibrium. Graphically, this means that the economy lives on the brown horizontal line labelled $\mathbf{a}^{RA}(r)$ in Figure 1 at all points in time. The real government budget constraint implies that $db_t = [\rho b_t - s^*]dt$. It follows that real debt is increasing when it is above steady-state, and decreasing below steady-state, as illustrated by the arrows in Figure 1. Paths with increasing debt are ruled out as equilibria by showing that they violate a household transversality condition. Paths in which debt is decreasing are ruled out since they violate the household's borrowing constraint in finite time. This argument is formalized in Online Appendix A. It follows that the stationary equilibrium is the unique real equilibrium and the initial price level and subsequent inflation are uniquely determined:

$$P_0 = \frac{B_0}{b^{RA}} \qquad \text{and} \qquad \pi_t^{RA} = i^* - \rho - g$$

Equilibrium paths display an initial jump in the price level at $t = 0$, and a constant inflation rate equal to steady-state inflation for $t > 0$.

Uniqueness in Representative Agent Economies with Bonds-In-Utility. The heterogeneous agent economy differs from the representative agent economy in part because the steady-state asset demand function is not perfectly elastic. In Online Appendix B we describe a simple representative agent economy in which households directly generate utility by holding real government debt. This economy features a steady-state asset demand function $\mathbf{a}^{BIU}(r)$ that has the same qualitative properties as $\mathbf{a}(r)$. In this economy, all equilibria lie on the one-dimensional manifold $\mathbf{a}^{BIU}(r)$ at all points in time, and away from steady-state the dynamics of government debt are unstable. A transversality condition and borrowing constraint rule out explosive paths in either direction as equilibria and hence the steady-state equilibrium is the unique equilibrium. The initial price level and subsequent inflation are uniquely determined. With positive primary surpluses, the difference between this economy and the standard representative agent economy is that the real interest rate is endogenous and depends on the level of surpluses. See Online Appendix B.3 for a formal argument.

State-Space Representation for Heterogeneous Agent Economy. Establishing that there is no multiplicity of non-stationary equilibria in the heterogeneous agent economy is more difficult than in the representative agent bonds-in-utility economy because the equilibria do not lie on a one-dimensional manifold. The aggregate state for the heterogeneous agent economy consists of the household asset and endowment distribution $g_t(a, z)$.[15] It is useful to partition this distribution into two components,

[15] The absence of the interest rate r_t from the aggregate state is not immediately obvious. However, as we verify below, in equilibrium it is implied by the joint distribution $g_t(a, z)$. In our quantitative analysis, we consider unanticipated time-varying shocks to various exogenous parameters. In these

which we denote by $\Omega_t := \{f_t(\omega, z), b_t\}$

(i) $f_t(\omega, z)$: the joint distribution of household asset shares and endowment shares

(ii) b_t: the level of real government debt.

The reason for partitioning the aggregate state in this way is that the two components have different dynamic properties. The distribution $f_t(\omega, z)$ is backward-looking and cannot jump. The level of real debt is a jump variable. It can jump because different values of the initial price level P_0 revalue the outstanding stock of nominal bonds B_0. Partitioning in this way makes it clear that although the household distribution $g_0(a, z)$ can jump, it can only jump along a single dimension such that the relative wealth holdings of each household remains unchanged. Using this state variable, we can write the consumption function $c_t(a, z)$ as $c(a, z, \Omega_t)$, where dependence on time is completely subsumed in the aggregate state.

Roadmap. Our discussion of uniqueness involves two steps. First, we show that any paths for b_t that diverge in either direction are not consistent with equilibrium because they involve eventual violation of either the borrowing constraint or a necessary household transversality condition. Second, we argue that the dynamics of Ω_t around the unique stationary equilibrium are locally saddle-path stable. Given an initial distribution $f_0(\omega, z)$ in the vicinity of $f^*(\omega, z)$, saddle-path stability implies that there is a unique initial value for the jump variable b_0 and unique subsequent paths of the aggregate state Ω_t such that the economy converges to $\Omega^* = \{f^*(\omega, z), b^*\}$.[16]

Ruling Out Explosive Equilibria. In Online Appendix C.3, we show that all paths of government debt b_t that grow at rate $r_t < \rho$ imply eventual violation of the following household transversality condition:

$$\lim_{T\to\infty} \mathbb{E}_{jt}\left[e^{-\rho T} c_T(a_{jT}, z_{jT})^{-\gamma} a_{jT}\right] \leq 0. \tag{24}$$

cases, the state space Ω_t needs to be expanded to include the law of motion for these exogenous driving processes.

[16]We must also rule out the possibility of non-stationary equilibria that remain bounded away from the stationary steady-state and involve cycles or similar dynamics. Although we cannot prove that no such equilibria exist, we have not encountered any numerically.

and hence cannot be part of aequilibrium.[17] Sufficient conditions for the equilibrium interest rate r_t in the heterogeneous agent economy to be below the discount rate ρ for all $t \geq 0$ are established in Not For Publication Appendix G.

Useful Characterization of Equilibrium Real Rate. In Online Appendix C.1 we derive expressions for expected consumption growth $\mathbb{E}_t[dc_{jt}]$ for constrained and unconstrained households. Here we use the short-hand notation $c_{jt} := c(a_{jt}, z_{jt}, \Omega_t)$ to denote the consumption of household j at time t. By aggregating these expressions across households, applying the law of iterated expectations, and imposing market clearing we derive the following relationship between the real rate and the aggregate state Ω_t,

$$0 = \underbrace{\frac{\mathcal{C}_t^u}{\gamma}(r_t - \rho)}_{\text{intertemporal substitution}} + \underbrace{\frac{\mathcal{C}_t^u}{\gamma}\tilde{\mathbb{E}}_t^u\left[\sum_{z'} \lambda_{z_j,z'}\left(\frac{c(\omega_j, z', \Omega_t)}{c_{jt}}\right)^{-\gamma}\right]}_{\text{precautionary motive}} + \underbrace{\mathbb{E}_t\left[\sum_{z'} \lambda_{z_j z'}\left\{c(\omega_j, z', \Omega_t) - c_{jt}\right\}\right]}_{\text{intertemporal smoothing}} \tag{25}$$

The expectation operator $\tilde{\mathbb{E}}_t^u$ is a consumption-weighted mean across the set of unconstrained households, and $\mathcal{C}_t^u$ is the total consumption of unconstrained agents. Not For Publication Appendix F contains a full derivation of this relationship.[18]

Equation (25) can be interpreted as balancing three forces driving changes in aggregate consumption that must net out to zero in an endowment economy. The first term is an intertemporal substitution motive for saving. The second term is the average precautionary savings motive. The presence of $\mathcal{C}_t^u$ captures the fact that this saving motive is only active for unconstrained households. The final term reflects an intertemporal motive for smoothing income shocks. In equilibrium, the interest rate is set so that the negative intertemporal substitution motive exactly offsets the combined effects of the precautionary saving and intertemporal smoothing motives.[19]

Equation (25) also confirms that the real rate is not required as a separate component of the aggregate state since that equation implicitly defines a time-invariant

[17]Establishing the transversality condition (24) as a necessary condition for household optimality is non-trivial. Kamihigashi (2001) shows that it is necessary in an analogous deterministic economy. Kamihigashi (2003) shows necessity in a discrete time stochastic economy.

[18]Not For Publication Appendix G contains the analogous formula for the real rate functional when idiosyncratic endowments follow a diffusion process.

[19]In the special case with quadratic utility, no borrowing constraints (hence, no precautionary saving) and $r_t = \rho$, equation (25) states that consumption is a martingale.

functional from Ω_t to r_t that holds at all times in equilibrium:

$$r_t = \mathbf{r}\left[\Omega_t\right]. \tag{26}$$

Local Saddle Path Stability. We derive the dynamics of the the aggregate state Ω_t by expressing the Kolmogorov Forward Equation (13) in terms of asset shares, and substituting the real rate functional (25) into the government budget constraint (18):

$$\begin{aligned}
\partial_t f_t(\omega, z) &= -\partial_\omega \left[f_t(\omega, z) \frac{1}{b_t} \left\{ z - \tau^*(z) - c(\omega b_t, z, \Omega_t) + s^* \omega \right\} \right] && (27) \\
&\quad - f_t(\omega, z) \sum_{z' \neq z} \lambda_{zz'} + \sum_{z' \neq z} \lambda_{z'z} f_t(\omega, z') \\
\frac{\mathrm{d}b_t}{\mathrm{d}t} &= \mathbf{r}\left[\Omega_t\right] b_t - s^* && (28)
\end{aligned}$$

Since this system is comprised of a one-dimensional jump component b_t and an infinite dimensional backward looking component $f_t(\omega, z)$, local saddle-path stability requires that, around the steady-state, this PDE system has one positive eigenvalue and non-positive remaining eigenvalues.

Discretized Economy. Although we are not able to prove saddle-path stability for the full continuum economy, we have found the system to be saddle path stable in our numerical explorations of discretized versions of this economy. Here we offer some intuition for local saddle-path stability from this discretized economy.

We consider a discrete approximation to $f(\omega, z)$ on a grid for relative asset shares of size N_ω, which we denote by the $N \times 1$ vector $\mathfrak{f}$ where $N = N_\omega \times N_z$. In Online Appendix C.4 we show that the finite difference approximation the PDE system (27) is given by the system of $N+1$ ODEs

$$\begin{aligned}
\frac{\mathrm{d}\mathfrak{f}}{\mathrm{d}t} &= \mathbf{A}_\omega \left[\mathfrak{f}_t, b_t\right]^T \mathfrak{f}_t + \mathbf{A}_z^T \mathfrak{f}_t && (29) \\
\frac{\mathrm{d}b}{\mathrm{d}t} &= \mathbf{r}\left[\mathfrak{f}_t, b_t\right] b_t - s^* && (30)
\end{aligned}$$

The matrices $\mathbf{A}_\omega \left[\mathfrak{f}_t, b_t\right]^T$ and $\mathbf{A}_z^T$ are upwind finite difference approximations to the two linear operators that comprise the KFE for (ω, z).[20]

[20] The transposes reflect the fact that these matrices are constructed by first constructing finite difference approximations to the adjoint operators in (27).

The dependence of $\mathbf{A}_\omega\left[\mathfrak{f}_t, b_t\right]^T$ on the distribution $\mathfrak{f}_t$ and real debt b_t arises for three reasons. First, a change in aggregate wealth b_t has a common effect on the interest earnings at all points in the wealth distribution. This direct effect is reflected by the b_t in the denominator of the top line of (27). Second, a change in aggregate wealth impacts consumption of all households via a wealth effect. This is reflected in the b_t in the first argument of the consumption function in (27). Finally, there are further general equilibrium effects on consumption because of future interest rate dynamics. These are reflected in the dependence of the consumption function on the aggregate state Ω_t in the third argument.

In Online Appendix C.4, we linearize the discretized system (29) around the steady state $(\mathfrak{f}^*, b^*)$ and show that the local dynamics are approximately

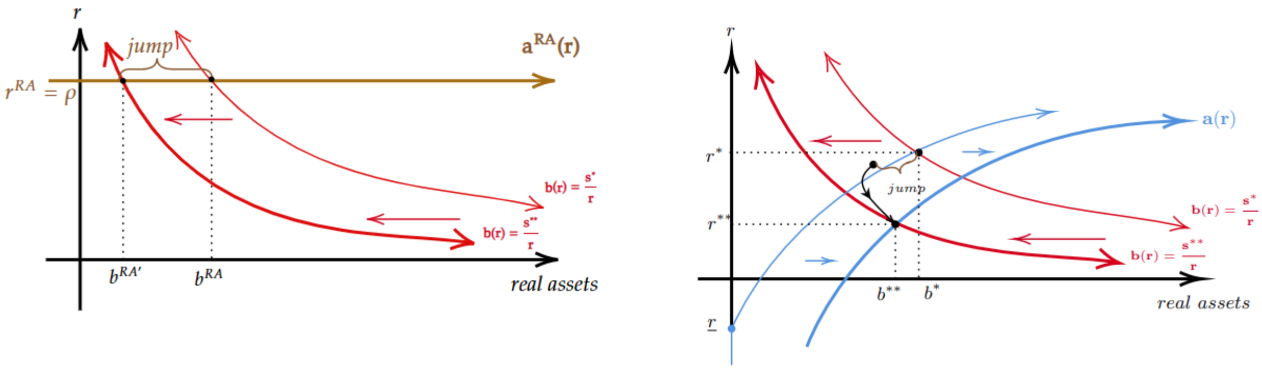
$$\begin{pmatrix} \frac{\mathrm{d}\mathfrak{f}}{\mathrm{d}t} \\ \frac{\mathrm{d}b}{\mathrm{d}t} \end{pmatrix} \approx \begin{pmatrix} \mathbf{A}_\omega^{*T} + \mathbf{A}_z^T & \nabla_b \mathbf{A}_\omega^T\left[\mathfrak{f}^*, b^*\right] \\ 0 & b^*\left\{\partial_b \mathbf{r}\left[\mathfrak{f}^*, b^*\right] - \left(-\frac{r^*}{b^*}\right)\right\} \end{pmatrix} \begin{pmatrix} \mathfrak{f}_t - \mathfrak{f}^* \\ b_t - b^* \end{pmatrix} \tag{31}$$

where term $\nabla_b \mathbf{A}_\omega^T\left[\mathfrak{f}^*, b^*\right]$ is the $N_\omega \times 1$ vector of derivatives of $\mathbf{A}_\omega^{*T}$ with respect to real debt b.

The approximation in (31) refers to the zero in the bottom left element of the Jacobian. Our approximation requires this term to be small only relative to the term in the bottom right element of the Jacobian. This means we require that around the steady-state, the dynamics of real government debt are more sensitive to changes in the *level* of real debt, holding the distribution of asset shares constant, than to changes in the *distribution* of asset shares, holding the level of real debt constant.[21]

In this case, the Jacobian is approximately block triangular, allowing us to sign the eigenvalues of the full system: $\mathbf{A}_\omega^{*T} + \mathbf{A}_z{}^T$ is an irreducible transition rate matrix and so has a single zero eigenvalue and remaining negative eigenvalues. The sign of the remaining eigenvalue is given by the sign of $\partial_b \mathbf{r}\left[\mathfrak{f}^*, b^*\right] b^* + \mathbf{r}\left[\mathfrak{f}^*, b^*\right]$. The first term is the inverse of the derivative of the steady-state household asset demand curve, multiplied by the level of steady-state assets. The second term is the steady-state interest rate. As both terms are positive under constant positive surpluses, the remaining eigenvalue

[21]This assumption might appear at odds with our substantive messages that emphasize changes in the distribution of real wealth as a quantitatively important factor in driving inflation and price level dynamics. However, as our simulations confirm, these are not contradictory: the feedback from the distribution of shares to the debt dynamics are large enough to be quantitatively meaningful, but would need to be orders of magnitude larger to alter the qualitative features of the dynamic system.



(a) Representative Agent Economy **(b)** Heterogeneous Agent Economy

Figure 2: A permanent reduction in surpluses

is strictly positive and the economy saddle-path stable.

3.3 Example: Permanent Reduction in Surpluses

We use a permanent reduction in surpluses as an example to illustrate the saddle-path dynamics. Consider a fiscal authority that unexpectedly changes the tax function from $\tau^*(z)$ to $\tau^{**}(z) = (1 - \Delta_s)\tau^*(z)$ so that primary surpluses decline to $s^{**} = (1 - \Delta_s)s^*$, with $\Delta_s \in (0, 1)$. The new steady-state government bond supply function is $\mathbf{b}(r) = \frac{s^{**}}{r}$, which is displayed as a leftward shift of the red line in Figure 2.

First, consider the effects of this change in the representative agent economy. The initial steady-state equilibrium before the change is indicated by b^{RA}. When the level of surpluses fall, the economy immediately jumps to the new steady-state equilibrium at the point labelled $b^{RA'}$. The level of real debt immediately falls to $(1 - \Delta_s)b^{RA}$, which is achieved by a one-time upward jump in the price level from P_0 to $\frac{P_0}{1-\Delta_s}$ with no change in either the real interest rate or inflation. The stock of nominal debt is unchanged, but real surpluses are reduced and thus the price level must jump to lower the real value of outstanding debt.

In the heterogeneous agent economy, the initial steady-state equilibrium is indicated by the point (b^*, r^*). In contrast to the RA model, a change in the tax and transfer function induces a shift in the steady-state household asset demand function for two reasons: (i) it affects disposable income; and (ii) it alters the degree of risk-sharing in the economy. In this example, the effect is to shift the $\mathbf{a}(r)$ curve to the right. The new steady-state after the change is indicated by the point (b^{**}, r^{**}).

Unlike in the representative agent economy, the economy does not jump immediately to the new steady-state. Rather, saddle-path dynamics imply that on impact of the change there is a one-time jump in the level of real debt to the unique value of b_0 that is consistent with non-explosive dynamics, which then determines a unique r_0 through the real rate functional (25). This is indicated by the leftward jump in Figure 2b. The initial jump is achieved by a rise in the price level that devalues all households' wealth proportionately.[22] This shift in the wealth distribution then induces trading among households as the interest rate falls smoothly to its new steady-state level. Without any change in monetary policy, inflation rises smoothly during this transition until it reaches its new steady-state level, which is higher than in the original steady state by the amount $r^{**} - r^*$.

4 Primary Deficits $s^* < 0$

We now assume that the fiscal authority runs a constant primary deficit. We first show that there are zero or two steady-state equilibria, depending on the level of deficits. We then characterize the out of steady-state dynamics and non-stationary real equilibria. We end this section with a discussion of alternative ways to restore uniqueness of a saddle-path stable equilibrium and hence a unique path for prices.[23]

4.1 Stationary Equilibria

The household asset demand $\mathbf{a}(r)$ function is qualitatively unchanged with $s^* < 0$. However, the steady-states of the government budget constraint, $\mathbf{b}(r) = s^*/r$ is an upward-sloping hyperbola for $b_t \geq 0$, as depicted in Figure 3. Note that with $s^* < 0$, any steady-state equilibria must have a real rate that is below the growth rate of the economy $r^* < 0$. From Figure 3, it is immediate that if such a steady-state equilibrium exists, then generically there will be two steady-state equilibria, as indicated by the two intersections of the asset supply and demand curves.[24] For a given nominal

[22] In general, the initial jump in the price level may undershoot or overshoot its long-run value depending on the nature of the transfer function.

[23] In Online Appendix C.5 we consider the case where s^*=0. Like in the case with $s^* > 0$, there is a unique equilibrium with a finite price level and the path of prices is uniquely determined. The steady-state real interest rate and real assets are $r^* = 0$ and $b^* = \mathbf{a}(0)$, respectively.

[24] This conclusion follows from the existence of a $\underline{r}$ such that for all $r_t < \underline{r}$, households do not save, meaning that the household steady-state asset demand curve intersects the $b = 0$ axis at a finite

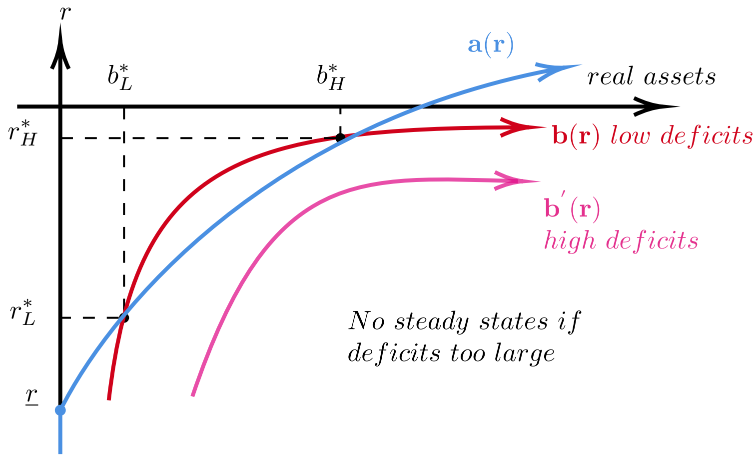


Figure 3: Maximum steady-state deficits

interest rate, the top equilibrium (b_H^*, r_H^*) has a higher level of real debt, higher real interest rate and lower inflation than the bottom equilibrium (b_L^*, r_L^*). In Online Appendix C.6 we show that the high interest rate steady-state Pareto dominates the low interest rate one by reducing the volatility of individual consumption growth.

Maximum Deficits. There exists a maximum level of deficits that is consistent with the existence of a stationary equilibrium where the price level is finite and government debt is valued. As the level of deficits increases, the government asset supply curve shifts downward to the right, as illustrated in Figure 3. The maximum deficit is attained when the asset supply and demand curves are tangent to each other, which occurs at the point where the interest-rate elasticity of the steady-state household asset demand curve is equal to unity: $\mathbf{a}'(r)r/\mathbf{a}(r) = -1$.

This condition reflects the fact that the maximum attainable level of deficits depends on the strength of households' desire to hold assets for precautionary reasons. It follows that a change in the nature of after-tax idiosyncratic endowment risk can shift the asset demand curve $\mathbf{a}(r)$ and hence the maximum deficit. Any reduction in s^* must be implemented via a change in the function $\tau^*(z)$. Depending on the change in progressivity, the maximum deficit may increase or decrease through a shift in $\mathbf{a}(r)$. In general, a change in the tax function that reduces the amount of uninsured risk will lower the maximum attainable deficit because households have less incentive to accumulate precautionary savings.[25] In Section 5 we use our calibrated model to

interest rate $\underline{r}$. This discussion maintains the assumptions outlined in Section 3.1 so that $\mathbf{a}(r)$ is monotonically increasing. Without these assumptions, there is generically an even number of steady states.

[25] Amol and Luttmer (2022) also emphasize that fiscal space depends on the overall level of risk

illustrate these forces.

Non-uniqueness of Price Level and Inflation. Since there are two steady-state equilibria with $s^* < 0$,standard FTPL arguments for uniqueness of the price level do not hold. Additional assumptions on fiscal policy must be imposed, or other modifications made to the economy, in order to uniquely pin down the price level and inflation. We discuss these possibilities in Section 4.3, but first we characterize the set of non-stationary equilibria.

4.2 Non-stationary Equilibria

Local Dynamics. We can characterize the local dynamics around each of the two steady states following the same line of argument as we did for the case with $s^* > 0$. The dynamics obey the same PDE system (27). The arguments we gave for why the eigenvalues associated with the backward looking component $f(\omega, z)$ are all non-negative remain unchanged. As before, we sign the eigenvalue associated with the jump variable b_t by assuming that – in the vicinity of a steady-state equilibrium – the effect on government debt dynamics due to general equilibrium feedback from movements in the distribution are small relative to the overall effect of changes in interest payments:

$$\frac{db_t}{dt} \approx b^* \left\{ \partial_b \mathbf{r} \left[\mathrm{f}^*, b^* \right] - \left(\frac{r^*}{b^*} \right) \right\} \tag{32}$$

The term in braces is the difference between the slopes of the steady-state asset demand function ($\partial_b \mathbf{r}[\Omega^*] = (\partial_r \mathbf{a}[r^*])^{-1}$) and the steady-state bond supply function ($-\frac{r^*}{b^*} = (\partial_r \mathbf{b}[r^*])^{-1}$). The eigenvalue associated with government debt b_t is therefore positive at the top steady-state, where the asset demand function crosses the asset supply function from below, and is negative at the bottom steady-state, where the asset demand function crosses the asset supply function from above. Hence the local dynamics around the top steady-state are saddle-path stable, similarly to the unique steady-state in the case with surpluses. The dynamics around the bottom steady-state are locally stable. Simulations confirm these properties.

Figure 4 illustrates these dynamics. For a given initial distribution $f_0(\omega, z) \neq f^*(\omega, z)$, there is a unique equilibrium converging to (b_H^*, r_H^*) and a continuum of equilibria converging to (b_L^*, r_L^*), indexed by the initial level of real debt b_0. Conse-

in an economy in which households face idiosyncratic shocks to their returns on capital.

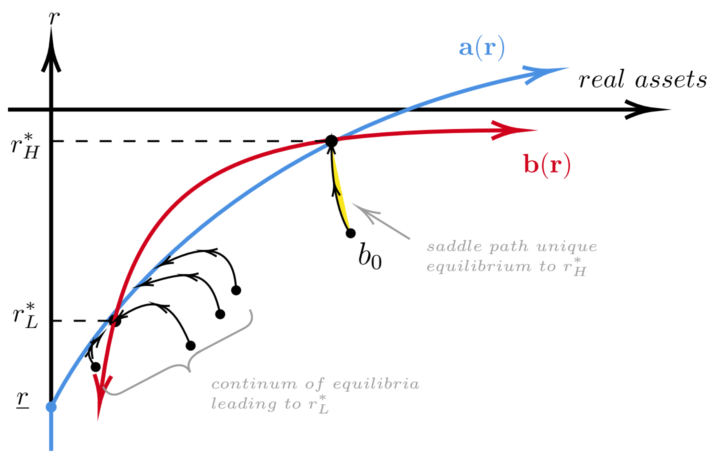


Figure 4: Non-stationary equilibria with deficits. For a given $f_0(\omega, z) \neq f^*(\omega, z)$, there are a continuum of equilibria indexed by initial real government debt.

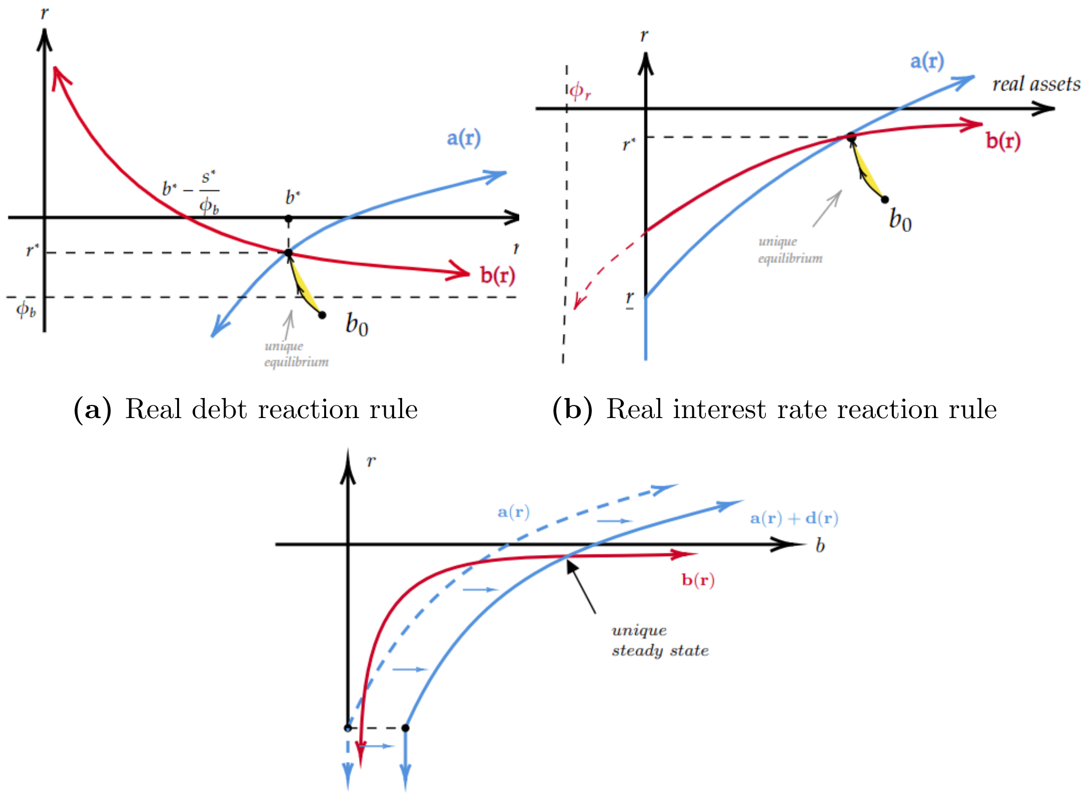
quently, the price level and inflation are not pinned down without additional assumptions that rule out almost all of these equilibria. Because it is the top equilibrium that is saddle-path stable, there is a lower bound on the initial price level that is consistent with equilibrium. This minimum initial price level is given by $P_0 = \frac{B_0}{b_0}$, where b_0 is the unique initial value of real debt for which the economy converges to the top saddle-path stable equilibrium.

Exact Characterization in a Bonds-In-Utility Economy. In Online Appendix B.4 we show that the representative agent economy with bonds in the utility function has qualitative steady-state properties that are the same as in the heterogeneous agent economy. In that economy, $\mathbf{a}^{BIU}[r]$ can be derived in closed form, and we can fully characterize the global dynamics: the top steady-state is unstable, the bottom steady-state is stable and there is a lower bound on the initial price level.

4.3 Options for Price Level Determination

Multiplicity of equilibria poses a challenge for quantitative work. We show that there are several ways to eliminate the locally stable steady-state and achieve uniqueness. First, through certain fiscal policy rules. Second, by introducing a foreign sector with relatively inelastic demand for domestic government debt. Lastly, through a form of long-run inflation anchoring.

Real Debt Reaction Rule. Until now we have assumed a fiscal rule that keeps primary deficits constant. Assume instead that the fiscal authority follows a rule in



(a) Real debt reaction rule **(b)** Real interest rate reaction rule

(c) Foreign demand

Figure 5: Alternative approaches to deliver a unique equilibrium with deficits

which primary deficits respond to real debt deviations from the steady-state level b^*:

$$s_t = s^* + \phi_b (b_t - b^*) \, . \tag{33}$$

The steady-state level of deficits is denoted by $s^* < 0$. Outside of steady-state, the fiscal authority varies deficits by changing the tax and transfer function $\tau_t(z)$. The steady-state government asset supply curve is given by $r = \phi_b + \frac{s^* - \phi_b b^*}{b}$. If $\phi_b < r^* < 0$, then for $b > 0$, this is a downward sloping curve that intersects the household asset demand curve only once, as illustrated in Figure 5a.[26] There exists a unique steady-state equilibrium which is saddle-path stable and hence the initial price level and subsequent inflation are uniquely determined. Online Appendix C.7 contains details. Note that the condition $\phi_b < r^*$ implies that when outstanding

[26]The household asset demand curve will also be affected, since higher levels of debt are associated with different transfer functions, which may alter the shape of the asset demand curve. In practice this effect can be made small by changing the level of deficits in an approximately distributional neutral way.

debt falls below its steady-state level, the government responds by cutting primary deficits. This reaction has a destabilizing effect on the debt accumulation process, which eliminates the bottom (stable) steady-state (b_L^*, r_H^*).[27]

Real Rate Reaction Rule. An alternative fiscal rule that also eliminates the stable steady-state equilibria is one in which primary deficits respond to deviations of the equilibrium real rate from its steady state,

$$s_t = s^* + \phi_r (r_t - r^*) . \tag{34}$$

In Online Appendix C.8, we show that a sufficient condition to eliminate the stable steady-state is $\phi_r < \frac{s^*}{r^* - \mathbf{a}^{-1}(0)} < 0$. Figure 5b illustrates this case. When the real rate falls below its steady-state value, the fiscal authority cuts primary deficits. This response has a destabilizing effect that eliminates the bottom stable steady-state.

Interest Payment Reaction Rule. We also consider a fiscal rule in which primary surpluses respond to deviations of real interest payments from their steady state level:

$$s_t = s^* + \phi_s (r_t b_t - s^*) . \tag{35}$$

In Online Appendix C.9 we show that the steady-state equilibria are unchanged from the baseline ($\phi_s = 0$). With an "active" rule ($\phi_s < 1$), the stability properties of the two steady-states are also unchanged. However, with a "passive" fiscal rule ($\phi_s > 1$), the stability properties of the two steady-states are reversed: the top steady-state is locally stable and the bottom one is saddle-path stable.

Inelastic Foreign Demand. If there is additional demand for government debt that is sufficiently interest-inelastic, for example from a foreign sector, then the bottom steady-state can be eliminated and uniqueness restored.

Denote the foreign demand for government debt as a function of the domestic real interest rate as $\mathbf{d}(r)$. The asset market clearing condition becomes $\mathbf{a}(r) + \mathbf{d}(r) = \mathbf{b}(r)$. To clearly see the effect of additional foreign demand, assume that it is perfectly

[27]This rule has the somewhat unappealing feature that when government debt rises above its steady-state level, the government responds by running even larger primary deficits. However, this property is not important for uniqueness; the role of the rule is to eliminate the stable equilibrium with *low* levels of government debt. Upward explosive dynamics are ruled out even with a constant deficit policy as explained in Section 3. For example an asymmetric policy, in which primary deficits respond only to reductions in government debt would suffice for uniqueness.

inelastic, so that $\mathbf{d}(r) = b^f$. The overall asset demand curve is shifted to the right and the bottom steady-state disappears, as illustrated in Figure 5c. In Online Appendix D we offer a microfoundation based on a representative agent foreign sector that has bonds-in-utility preferences. We show that an interest rate elasticity of demand below one is sufficient to ensure that the two curves intersect only once.

Long-Run Inflation Anchoring. The previous approaches to delivering a unique path of prices work by making assumptions that eliminate the high inflation stable steady-state, leaving only the low inflation saddle-path stable steady state. An alternative route to uniqueness is to instead eliminate all dynamic equilibria that lead to the high inflation steady-state, leaving only the unique equilibrium leading to the low inflation steady-state. In Not For Publication Appendix H, we show that a central bank that coordinates *long-run* inflation expectations can successfully pin down the inflation and the price level in the *short-run* under a constant deficit fiscal policy rule.

5 Quantitative Exercises with Persistent Deficits

In this section we describe various quantitative experiments for a calibrated version of the model with persistent deficits in order to illustrate the role of redistribution and precautionary saving in shaping price level dynamics.[28]

5.1 Model Extensions

We incorporate the following two extensions of the baseline model.

Extension I: Unsecured Household Credit. We allow for a non-zero borrowing limit. This permits nominal positions to be negative, thereby allowing some households to experience a positive wealth effect from an unanticipated rise in the price level, as in Doepke and Schneider (2006) and Auclert (2019). We assume that households can borrow up to a fixed limit that is denominated in real terms. We interpret it as unsecured borrowing, such as credit card debt, and impose an exogenous wedge between borrowing and saving rates. See Online Appendix E.1 for details.

[28] Our economy is a flexible price, endowment economy in continuous time. In reality, the price level does not jump. Rather, the initial bursts of inflation from these shocks are drawn out over a period of time. Despite this simplification, the general forces at work are informative about the two-way feedback between the equilibrium wealth distribution and movements in the price level.

Parameter	Value	Target
Preferences		
γ Inverse EIS	1	
ρ Discount rate	2.8% p.a.	debt-to-annual GDP ratio of 1.10
Income Process		
g Real output growth	2.0% p.a.	average growth rate post-war
λ Arrival rate of earnings shocks	1.0 p.a.	
σ St. Dev. of log quarterly earnings	1.2	
Household Borrowing		
$\underline{a}$ Borrowing limit	$\$15,000$	70% of quarterly household earnings
$r^b - r$ Borrowing wedge	16% p.a.	average rate on credit card debt
Tax and Transfers: $\tau(z) = \tau_0 - \tau_1 * z$		
τ_1 Proportional tax rate	30%	personal taxes / labor income
τ_0 Lump sum transfer	33.3% of GDP	deficit: $s^* = -3.3\%$
Government Debt		
δ Maturity rate of government debt	20% p.a.	average duration of 5 years
Monetary Policy		
i Nominal rate	1.5%	average Federal Fund Rate

Table 1: Calibrated parameter values and targets.

Extension II: Long-Term Debt. We assume that the government issues long-term debt with a constant maturity rate. The switch to long-term debt has no effect on the preceding analysis of price level determination. However, as shown by Sims (2011) and Cochrane (2018), debt duration plays a key role in the dynamics of inflation after unanticipated changes in the nominal interest rate. This mechanism surfaces in some of our experiments where we explore monetary policy rules beyond an interest rate peg. Online Appendix E.2 describes the model with long-term debt.

5.2 Parameterization

Preferences. We set the elasticity of inter-temporal substitution γ to 1 so that households have log utility. We choose the discount rate ρ to match an annual debt-to-GDP ratio of 1.10 in the low inflation steady state. This target, which corresponds to the debt-to-GDP ratio in US data for the years leading up to the pandemic (2014-2019), implies a calibrated annual discount rate of 2.8%.

Endowment Process. We assume an annual aggregate real growth rate g of 2%, which was the US per-capita average over the post-war period.[29] Idiosyncratic endowment shares follow an $N_z = 5$ state process, with switching rates chosen so that income shocks arrive on average once per year and the endowment process generates a standard deviation of log quarterly earnings of 1.08, in line with US micro data.[30]

Household Borrowing. We set the borrowing limit $\underline{a}$ to \$15,000, which is approximately 70% of average quarterly household earnings to match the median credit card limit for working-age population in the Survey of Consumer Finances (SCF) (Kaplan and Violante, 2014). We set the wedge between the interest rates on borrowing and saving to 16% p.a., based on typical interest rates on unsecured credit card debt.[31] Because of this exogenous wedge, the real borrowing rate is positive, and the natural borrowing limit is finite and exceeds the ad-hoc limit.

Tax and Transfer System. The tax and transfer system consists of a lump-sum transfer and proportional tax,

$$\tau(z) = -\tau_0 + \tau_1 z.$$

We set the proportional tax rate τ_1 to 30% to match the ratio of personal taxes and social insurance contributions to total labor income (NIPA Table 2.9) for 2014-2019. We then set the lump-sum transfer τ_0 at 33.3% of aggregate output to generate a primary deficit s^* of -3.3% of GDP, the average for the US over that period.[32]

Government Debt. We assume that 20% of outstanding government debt matures each year to match a weighted average duration of 5 years (US Treasury). Given our target debt-to-GDP ratio of 110%, and primary deficit of 3.3%, the implied steady-state real interest rate equals $\frac{s^*}{B^*} + g = \frac{-0.033}{1.1} + 0.02 = -1\%$ p.a.

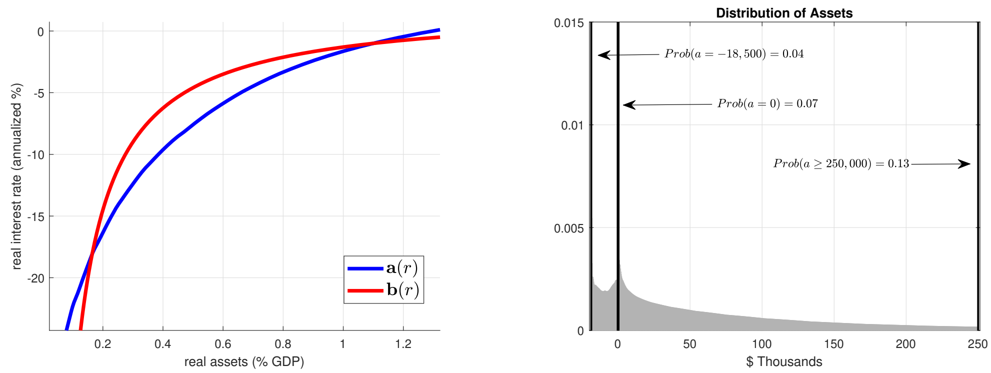
Monetary Policy. We assume that the central bank pegs the nominal rate at 1.5% p.a., consistent with the average interest rate target in the years leading up to the

[29] See Series A939RX0Q048SBEA_PC1 from FRED, Federal Reserve Bank of St. Louis, https://fred.stlouisfed.org.

[30] See, for example, the Global Repository of Income Dynamics (GRID), https://www.grid-database.org/.

[31] See Table Consumer Credit - G19, Federal Reserve Board, https://www.federalreserve.gov/releases/g19/current/.

[32] The data sources for debt and deficits are series GFDEGDQ188S and FYFSGDA188S from FRED.



(a) Calibrated steady-states **(b)** Wealth distribution

Figure 6: Calibrated steady-states and wealth distribution

pandemic. With a real interest rate of -1%, the implied annual inflation rate is 2.5%.

5.3 Properties of Steady States

Figure 6a displays the two stationary equilibria implied by our calibration. In line with our targets, the low inflation saddle-path stable steady-state has an annual debt-to-GDP ratio of 110% and an annual inflation rate of 2.5%. The high inflation steady-state has an annual debt-to-GDP ratio of 17.5%, and an annual inflation rate of around 19.5%. In what follows, we focus on the low-inflation steady state.

Wealth and MPC Distribution. Figure 6b and Table 2 illustrate that the model is broadly consistent with the distribution of liquid wealth in the 2019 SCF.[33] Expressed in 2019 dollars, mean and median household wealth in the model are \$116,000 and \$40,000 respectively. 19% of households have negative wealth and 27% of households have less than \$1,000. These moments were not targeted in our calibration, which was disciplined by aggregate statistics on national debt.

The average quarterly MPC in the model is around 14%, with the highest MPCs among the low-income households that either have close to zero wealth and so are near a kink in their budget constraint, or have substantial negative wealth and so are close to the borrowing limit.[34]

[33] Our definition of liquid wealth includes money market, checkings, savings, and call accounts, as well as directly held mutual funds, stocks and bonds, minus credit card and uncollateralized debt. We exclude the top 1% of households in the SCF by liquid wealth because of the well-known difficulties in matching the right-tail of the wealth distribution in this class of models.

[34] Not For Publication Appendix J contains additional details on the distributions of wealth and marginal propensities to consume in the model.

Table 2

Mean liquid assets	Data	Model
Mean assets	$116,000	$100,317
Frac. with $a <$ $0	20.67%	19%
Frac. with $a <$ $1,000	37%	27%

Note: Moments of the wealth distribution in the model and the data. Monetary values expressed in 2019 dollars. Data is from the 2019 Survey of Consumer Finances (SCF) with the top 1% of households by liquid wealth are excluded. See the main text for the definition of liquid assets in the data.

Maximum Sustainable Deficit. As discussed in Section 4.1, there exists a maximum possible level of permanent deficits consistent with existence of an equilibrium where debt is valued. The size of this maximum deficit depends on whether it is reached by expanding lump-sum transfers or cutting proportional taxes. Under our calibration, raising transfers yields a maximum deficit of 4.6% of output, a 39% increase from the baseline steady-state value of 3.3%. Instead, lowering taxes allows the government to run a maximum deficit of 4.8%, a 45% increase from the baseline.

Lower proportional tax rates are, in general, associated with higher maximum steady-state deficits because they increase the volatility of disposable earnings. Households therefore bear more uninsured idiosyncratic income risk which raises their overall precautionary demand for safe liquid assets. For a given interest rate r, households are willing to hold more government bonds if they bear more idiosyncratic risk, giving the government more room to expand its deficit. Graphically, a lower value for τ_1 induces an outward shift in the the steady-state household asset demand curve (recall Figure 3). The same logic, with signs reversed, applies to an expansion of lump-sum transfers because they reduce the volatility of net earnings.

The role of precautionary saving is quantitatively important. For example, in an extreme case without proportional taxes ($\tau_1 = 0\%$), the maximum sustainable deficit that can be achieved by expanding transfers is 9.5%, almost three times as large as in our baseline. For similar reasons, when households are prohibited from borrowing, the maximum sustainable deficit rises to 5.9%. A key lesson from these experiments is that reforms that loosen credit, make tax and transfer systems more progressive, or provide more insurance to households reduce future fiscal space available to the government. These reforms restrict the government's ability to expand deficits or cut surpluses, and therefore may constrain its ability to use expansionary fiscal policy to respond to adverse aggregate shocks.

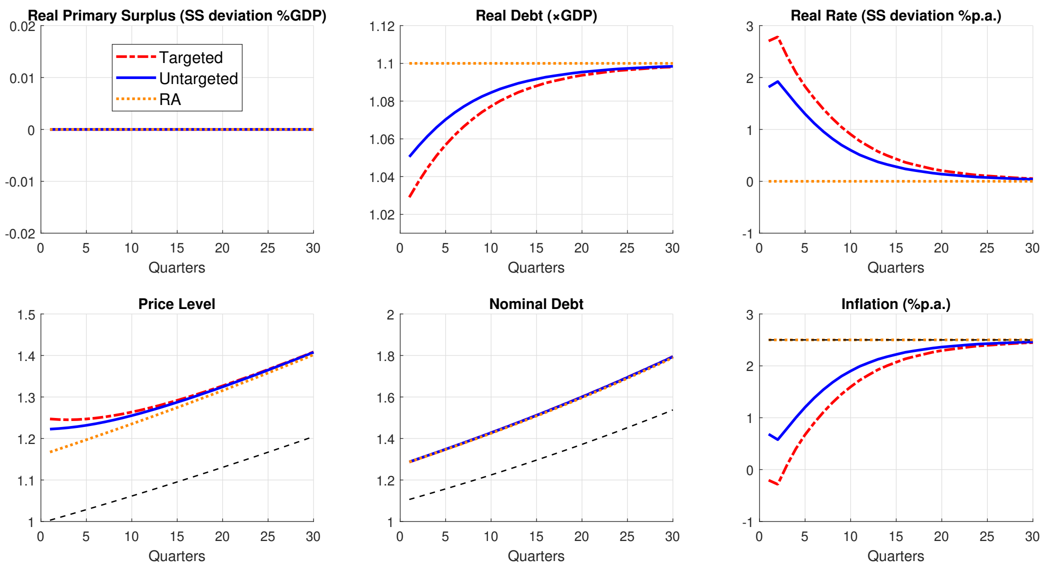
Implications for Secular Stagnation. A recent literature argues that the secular decline of real rates observed in the US and other developed economies is due to rising income risk and inequality, which has been accelerated by the sharp debt deleveraging that occurred after the 2008 financial crisis (Auclert and Rognlie, 2018; Eggertsson et al., 2019; Mian et al., 2021b). The argument is that higher inequality leads to a redistribution of income from the high-MPC poor to the low-MPC rich, which increases overall demand for wealth in the household sector. Similarly, more uninsured income risk or a tighter borrowing limit create a stronger precautionary motive, which increases demand for government bonds. These forces all manifest as an outward shift of the household asset demand function $\mathbf{a}(r)$. In a conventional economy with positive rates and permanent surpluses, such outward shifts in household asset demand indeed leads to a lower steady-state real rate.

However, in an economy with permanent deficits and a negative real rate, these comparative statics are reversed when the economy starts in the low-inflation steady state. An outward shift of the household asset demand function $\mathbf{a}(r)$ leads to a higher steady-state real rate. The reason is that in order to finance the same level of deficits with a higher quantity of debt, a less negative (i.e. higher) real rate is needed. This observation adds an important qualification to the commonly held view that shifts in the income distribution, income risk or deleveraging are candidate explanations for secular stagnation. In Section 5.5, we propose an alternative explanation for secular stagnation, rooted in the observation that in heterogeneous agent economies with persistent deficits and $r < g$, larger primary deficits depress the real rate.

5.4 Fiscal Helicopter Drop

Our first experiment is inspired by the experience of the US and other developed countries in the wake of the COVID-19 shock. In response to the disruptions caused by the pandemic, the US issued a large quantity of additional government debt and distributed much of the proceeds to households. We capture the core features of this fiscal helicopter drop by simulating an unexpected one-time issuance of nominal debt equal to 16% of initial outstanding government liabilities (equivalent to the observed 16% rise in the US debt-GDP ratio in 2020), which is distributed as a one-time lump-sum transfer to households. We consider two versions of this policy: one where transfers are distributed uniformly and one where transfers are distributed only to

Figure 7



Note: This figure plots impulse responses to a targeted and untargeted helicopter drop, aggregated at the quarterly frequency. The helicopter drop is a one-time issuance of 16% of total government nominal debt outstanding at $t = 0$. Only households in the bottom 60% of the wealth distribution receive the issuance in the targeted experiment (dashed red line). The orange line plots dynamics in the representative agent (RA) model. The dashed black line plots the initial steady state.

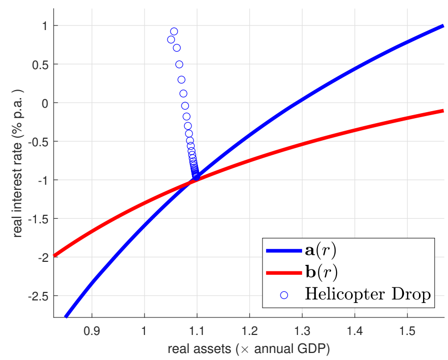
households in the bottom 60% of the wealth distribution, in line with the actual US experience.

Aggregate Effect of Fiscal Helicopter Drop. The effects of the fiscal helicopter drop are displayed in Figure 7. Since there are no changes to primary surpluses or any other structural parameters, the helicopter drop has no permanent real effects: the household and government nullclines are unchanged, and the economy converges back to its initial steady-state.

In the representative agent version of this economy, which is shown by the orange dotted line labelled "RA" in Figure 7, convergence is instantaneous.[35] The jump in the price level exactly offsets the new issuance of nominal debt so that the level of

[35]The representative agent economy is constructed to have the same steady-state debt-to-GDP ratio as in the heterogeneous agent economy. However, since the representative agent economy does not admit a steady-state with persistent deficits, we assume an annual surplus-to-GDP ratio of 3.3% and an equilibrium real rate of 1%. We adjust the nominal interest rate so that the inflation rate is the same in the two economies.

Figure 8



Note: This figure shows the computed saddle-path dynamics from a one-time issuance of nominal government debt in (r_t, b_t) space. The total issuance amounts to 16% of nominal government debt outstanding at $t = 0$. The blue dots depict quarterly aggregates.

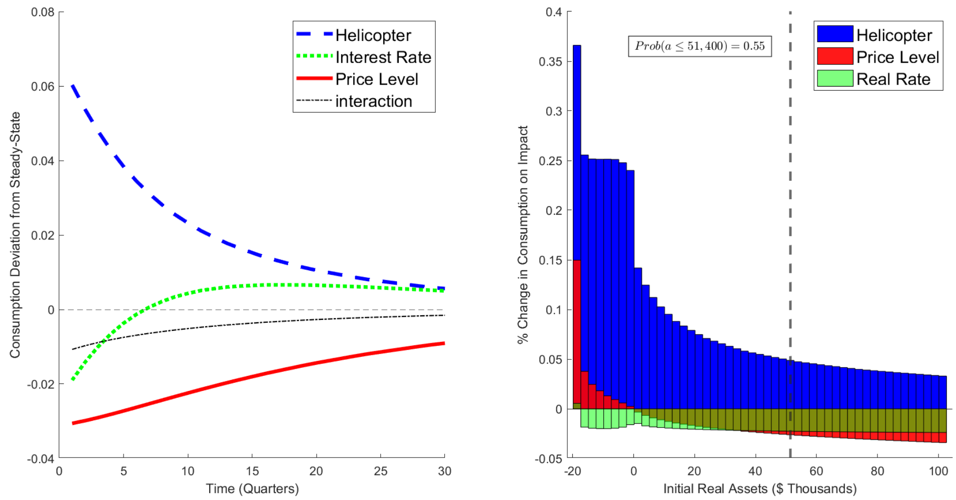
real debt remains constant and there are no further effects of the shocks.[36] However, n the heterogeneous-agent model, there are transitional dynamics. The computed saddle-path dynamics associated with this convergence in (r_t, b_t) space are displayed in Figure 8. The initial jump in the price level (bottom-left panel of Figure 7) is about 21%, higher than in the representative agent model, which more than offsets the 16% rise in nominal debt.

Why does an identical expansion in government debt place more upward pressure on the price level in the heterogeneous agent economy? The fiscal helicopter drop entails a redistribution of real wealth from high- to low-wealth households because the lump-sum transfer is progressive. Since the average MPC is higher among low wealth households, this redistribution raises the economy-wide desire to consume. With a constant aggregate endowment, the real interest rate must rise to restore goods market clearing. The higher (i.e. less negative) real interest payments require a reduction in total real government debt outstanding. Since nominal debt is fixed after the helicopter drop, the price level must then increase further. An alternative interpretation is simply that the additional spending pressure from redistribution, beyond the aggregate wealth effect, places more upward pressure on nominal prices than in the representative agent economy where only the wealth effect is present.

Decomposition of Fiscal Helicopter Drop. In addition to the the direct redistributive impact of the fiscal helicopter drop, there are two additional indirect

[36]The initial price jump in Figure 7 is slightly more than 16% because in this and other figures, we plot impulse response functions aggregated to a quarterly frequency.

Figure 9



Note: This figure decomposes the effect of the helicopter drop on consumption into its general equilibrium sub-components. The left panel depicts how each sub-component affects aggregate consumption over time in isolation. The right panel depicts the effect of each sub-component on initial consumption across the wealth distribution. The dashed black line on the right panel delineates households that experienced initial consumption gains and losses as a result of the helicopter drop in 2019 US dollars.

general equilibrium channels at play that shape the subsequent dynamics of the real rate and inflation. First, the upward jump in the price level redistributes wealth from savers to borrowers, and dilutes the real savings for households with a positive net nominal position. Second, the resulting rise in the real rate leads households to postpone consumption. The left panel in Figure 9 displays the dynamic effects of each of these channels on aggregate consumption. The helicopter drop itself raises consumption, while the higher price level lowers consumption. These effects diminish as the economy returns to steady-state. The higher real interest rate leads households to delay consumption, which is reflected by the initially lower but subsequently higher consumption in the green dotted line in Figure 9.

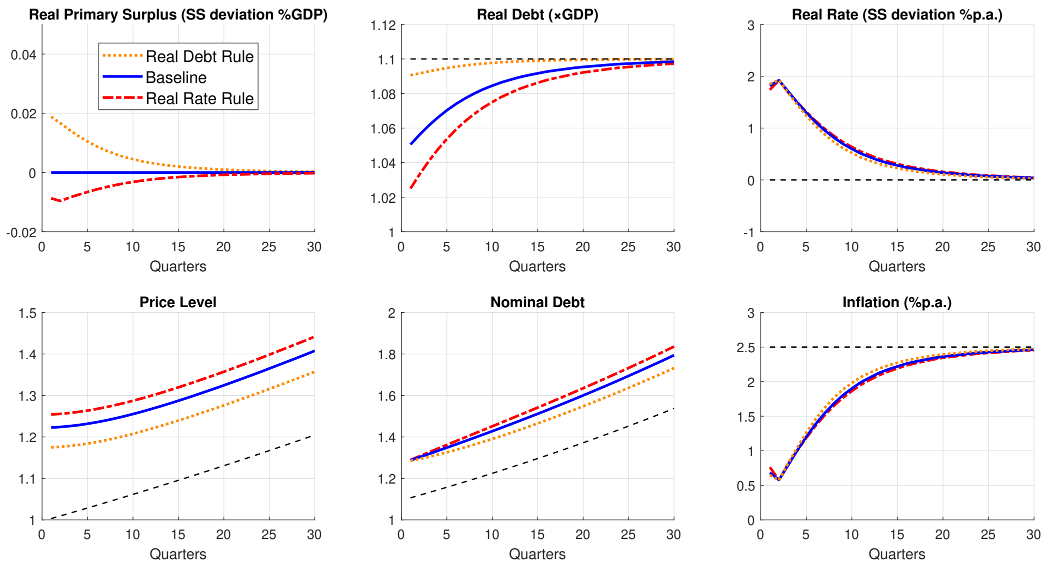
The aggregate decomposition masks substantial heterogeneity in the effect of these channels across households. The right panel of Figure 9 shows the contribution of each channel to the change in consumption on impact along the wealth distribution. Low-wealth households increase consumption substantially, predominantly due to their higher MPCs out of the direct helicopter drop at the steady-state price level. In addition, the jump in the price level induces households with negative wealth to increase their consumption, because it lowers the real value of their debt. For households with positive wealth, the higher price level reduces their consumption because the real value of their nominal savings is curtailed. The higher real interest rate weakens consumption for all households because of an intertemporal motive, except for households on the borrowing constraint. The dashed black line delineates the winners and losers of this experiment in terms of 2019 US dollars. Households with assets lower than $\$51,400$, which account for 55% of the population in our calibrated economy, gain from the helicopter drop.

Targeted vs Untargeted Fiscal Helicopter Drop. Figure 7 also shows that initial increase in the price level is even larger when the the helicopter drop is targeted towards poorer households. Compared to the untargeted case, the real interest rate rises by 1 additional percentage point on impact and, as a result, the price level jumps by an additional 4 percentage points (to 25%). In both the untargeted and targeted cases, the fiscal helicopter drop has a permanent effect on the price level and nominal government debt, but the inflationary effects are temporary. The saddle-path dynamics imply that both the real interest rate and the inflation rate return to their initial levels. In these experiments, the different price level responses between the heterogeneous agent and representative agent economies are mostly in terms of timing. The higher initial rise in prices in the heterogeneous agent economy is followed by lower inflation, and the long-run cumulative increase in the price level is the same in the two economies.

Fiscal Helicopter Drop Under Different Surplus Reaction Rules. To justify focusing attention on the saddle-path equilibrium we are implicitly appealing to long-run inflation anchoring. As discussed in Section 4.3, surplus reaction rules are an alternative route to uniqueness. Figure 10 shows that the price level, real rate and inflation dynamics from the fiscal helicopter drop are not sensitive to using either of the two classes of surplus reaction rules in equations (33) and (34) that guarantee a unique equilibrium.

However, the two rules differ in the direction that primary deficits respond to the fiscal helicopter drop. Under the real debt reaction rule (33), the downward revaluation of real debt from the initial burst of inflation leads the fiscal authority to cut deficits following the helicopter drop. Under the real rate reaction rule, the

Figure 10



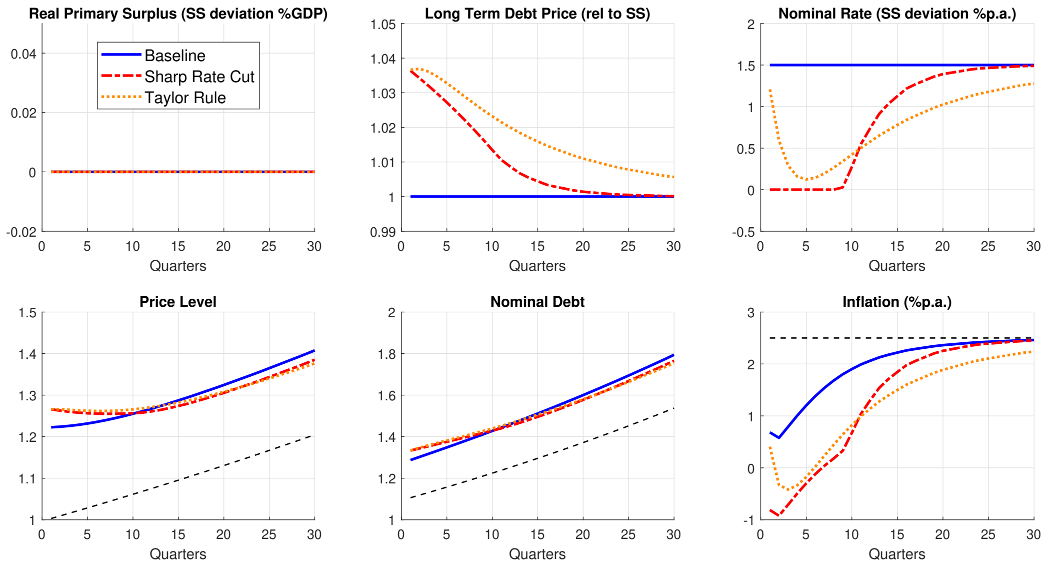
Note: Impulse responses to targeted fiscal helicopter drop under alternative fiscal rules. The dotted orange line corresponds to the "real debt rule" of equation equations (33) and the dashed red line corresponds to the "real rate rule" in equation (34) with parameter values of $\phi_b = -0.5$ and $\phi_r = -2$, respectively. The dashed black line plots the initial steady state.

higher real interest rate leads to a temporary increase in primary deficits.[37]

Fiscal Helicopter Drop Under Different Monetary Responses. Throughout our previous simulations we have assumed that the central bank holds the nominal rate constant at 1.5% in response to the helicopter drop. Figure 11 reports results from two alternative experiments in which nominal rates are lowered at the same time as the fiscal expansion, like was done by central banks around the world in 2020. The dotted orange line labelled "Taylor rule" shows the effects of following a lagged Taylor rule as in equation (22), with a feedback parameter $\theta_m = 1$ and a coefficient on inflation $\phi_m = 0.5$. The dashed red line labelled "sharp rate cut" shows the implication of an even more powerful monetary accommodation of the fiscal expansion, corresponding to an immediate cut in the short-term interest rate all the way to zero, followed by a gradual normalization after 9 quarters. For comparison, the blue line labelled "baseline" reproduces the dynamics holding the nominal rate constant.

[37] Cochrane (2023) argues that following an expansion in nominal debt, a reduction in primary deficits is more in line with the historical record for the U.S. However, Jacobson et al. (2023) discuss an important historical example in which new debt was issued with the explicit intention of generating inflation by committing to not raise future surpluses to repay the debt.

Figure 11

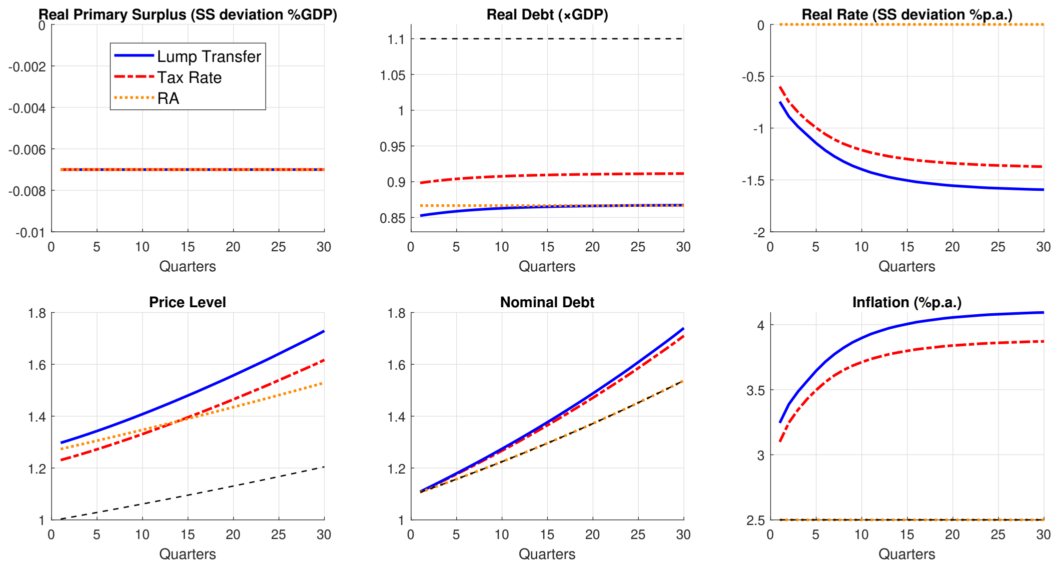


Note: Impulse response to targeted fiscal helicopter drop under different monetary policy responses. The dotted orange line corresponds to the Taylor rule in equation (22) with $\theta_m = 1$ and $\phi_m = 0.5$. The dashed red line is a temporary cut of nominal rates all the way to the zero lower bound. The dashed black line plots the initial steady state.

Monetary policy is a crucial driver of nominal aggregates. The behavior of long-term government bond prices is central to these dynamics. As explained in Sims (2011) and Cochrane (2018), a lower short-term nominal rate leads, through the yield curve, to a higher price of long-term government bonds. Thus, the overall price level must rise by a larger amount to achieve the same-size drop in the real value of outstanding government debt. Figure 11 shows that looser monetary policy causes an additional 4 to 6 percentage point increase in the price level upon impact, relative to the baseline with a nominal rate peg. The strength of this force is determined by the average duration of debt: the longer the duration, the bigger the initial jump in the price level. Different jumps in the price level, in turn, lead to different dynamics for real government debt and real interest rates through their effect on the real wealth distribution. However, we have found the effect on real variables to be quantitatively very similar across the three monetary specifications, provided that it is higher-wealth households that hold assets of longer duration.[38]

[38] If higher wealth households have longer duration portfolios, an unanticipated increase in monetary policy leads to larger capital losses for high-wealth households. However for moderate movements in the nominal rate, the relatively low MPCs of these households lead to small movements in the real rate. The assumption that high-wealth households hold relatively higher duration assets is

Figure 12



Note: Impulse response to a permanent expansion in primary deficits. The dotted orange line shows the effects of a reduction in surplus in the Representative Agent model. The blue line labelled "Lump Sum" illustrates the dynamics following an expansion of lump sum transfers. The dashed red line labelled "Tax Rate" plots dynamics following a tax cut. The orange line plots dynamics in the representative agent (RA) model. The dashed black line plots the initial steady state.

5.5 Permanent Deficit Expansion

Figure 12 displays impulse responses to a permanent deficit expansion from 3.3% to 4% of GDP. We consider two alternative policies for achieving a higher level of deficits. The solid blue line labeled "Lump-Sum" keeps the tax rate the same and raises the lump-sum transfer. The dashed red line labeled "Tax Rate" reduces the proportional tax rate, while keeping lump-sum transfers at their initial level.

As was shown in Figure 3, a permanent increase in deficits shifts the steady-state government nullcline downwards and to the right. Starting from the high real rate, low inflation steady-state, the long-run impact of the deficit expansion is to permanently lower both the real rate and the real value of government debt. These effects can be seen in the top row of Figure 12. The reduction in the value of real debt is achieved through a jump in the price level. In addition, because monetary policy does not respond, the lower real-rate translates into a permanently higher inflation rate. To prevent the permanent increase in deficits from leading to permanently

consistent with empirical evidence. See Doepke and Schneider (2006); Greenwald et al. (2021).

higher inflation, the central bank would need to track the fall in the real rate by decreasing its nominal rate target.

Hence in the heterogeneous agent economy with deficits and negative real rates, a secular increase in primary deficits can account for a secular decline in real rates, i.e. secular stagnation. The fact that permanently higher deficits result in a permanently lower real rate and higher inflation is a distinguishing feature of the heterogeneous agent economy relative to the representative agent economy, in which a permanent increase in deficits has no impact on real rates or inflation.

These effects are all more pronounced when deficits are increased by raising lump-sum transfers than by lowering the proportional tax rate. The reason is that raising lump-sum transfers lowers the amount of uninsured idiosyncratic risk, thereby weakening the overall precautionary motive in the economy, while lowering proportional taxes raises the overall precautionary motive. Graphically, these differences manifest as different shifts in the household asset demand curve $\mathbf{a}(r)$.

5.6 Additional Quantitative Results

Inflationary Effects of Redistributive Wealth Taxes. In order to emphasize the inflationary effects that arise from redistribution, Not For Publication Appendix K considers purely redistributive shocks: one-time wealth taxes levied on the top 10% of the wealth distribution, the proceeds of which are redistributed lump-sum to the bottom 60%. Although these shocks do not entail any new issuance of government debt or any change in primary deficits, they do cause a prolonged period of inflation.

Endogenous Output. Not For Publication Appendix L studies a permanent change in primary deficits in an economy where households make a labor-leisure choice with endogenous output. This extension serves to demonstrate that none of the qualitative forces relating heterogeneity and precautionary savings to prices and inflation that we have emphasized depend on an endowment economy per se.

6 Conclusions

We extend the fiscal theory of the price level to a heterogeneous-agent incomplete-market economy with flexible prices. In contrast to its representative agent counterpart, this model can be used to study an environment in which the government runs persistent deficits and the real rate is below the aggregate growth rate of the economy. This configuration is a more accurate representation of the current state of affairs in many developed economies.

After showing that this model generically has two steady-states, we proposed a number of ways to obtain uniqueness for price level and inflation dynamics. Armed with uniqueness, we performed experiments that illustrate the forces at work in our model. The feature of our economy that accounts for different dynamics relative to its representative agent counterpart is the two-way feedback between price-level dynamics on the one hand, and redistribution and precautionary saving on the other. Redistribution and precautionary saving are also key determinants of the maximum deficit the economy can permanently sustain.

In on-going work we are extending this framework in two directions. The first is to include nominal rigidities, which gives rise to smoother price level dynamics. It also offers us the possibility to quantitatively confront the FTPL with the joint dynamics of inflation and output observed in the data, along the lines of what Bianchi et al. (2023) did in a representative agent model. The second is to extend our model to a two-asset economy with both low return nominal government bonds, and higher return real productive assets. Incorporating a two-asset household sector as in Kaplan et al. (2018) opens the door to a quantitative framework with a richer characterization of the possible assets through which households can save.

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